Quantum Harvests: Optimizing Agricultural Yield through QUBO-Driven Crop Selection and Allocation

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Abstract

Crop selection and allocation play crucial roles in maximizing agricultural yield and profitability. In this study, we propose a novel approach using a Quadratic Unconstrained Binary Optimization (QUBO) model to optimize crop selection and allocation in agricultural systems. The objective is to determine the optimal combination of crops that maximizes yield or profit while considering the interactions and costs associated with different crops. The QUBO model incorporates negative impacts or costs associated with selecting each crop and positive effects or synergies between different crop combinations. By formulating the problem as a QUBO model, we enable the use of quantum annealing or classical optimization techniques to find the optimal solution. The model's effectiveness is demonstrated through numerical experiments and case studies. Results show that the QUBO-based approach provides significant improvements in crop selection and allocation decisions compared to traditional methods. It takes into account the complex interactions between crops and considers the trade-offs between costs and synergies, resulting in more efficient and profitable agricultural systems. The proposed model offers a flexible and adaptable framework that can accommodate various crop types, growth requirements, market conditions, and constraints. It provides decision-makers in the agricultural sector with a powerful tool to optimize crop selection and allocation, ultimately increasing agricultural yield and profitability while promoting resource sustainability.

Keywords: crop selection, agricultural optimization, QUBO model, yield maximization

Introduction

Lucas (2014) argued that Agriculture is one of the most important and challenging sectors of the global economy, as it provides food, income, and livelihoods for billions of people, while facing the impacts of climate change, population growth, land degradation, water scarcity, and market volatility. To cope with these challenges, farmers and decision-makers need to make optimal choices about what crops to grow, where to grow them, and how much land and resources to allocate to each crop. These choices can have a significant impact on the yield and profit of a farm, as well as the environmental and social outcomes of agricultural production Parenthetical. Agriculture stands as a crucial pillar in a nation's developmental journey. According to Akhavizadegan et al. (2021), time-dependent parameter estimation is crucial for accurate crop modeling The strategic planning of crops plays a vital role in bolstering farmers' income and influencing import-export policies within the agricultural sector. To address this, predictive techniques leveraging machine and deep learning are employed to recommend crop types for specific fields. Accurate crop predictions with minimal errors have posed significant challenges in recent times. (United Nations, nd., paragraph 4) Crop models offer the potential to expand our comprehension of how crops interact with agronomic practices over varying spatial and temporal dimensions. These models have gained recognition as valuable tools that aid agronomists, farmers, policymakers, and fellow researchers in making well-informed decisions and recommendations. Within this manuscript, we present a comprehensive guide on constructing a process-based crop model integrated within a broader cropping system framework. It is worth noting that various types of crop models exist, and therefore, we aim to clarify the specific focus of this paper. It's important to emphasize that this paper does not pertain to 3D-crop modeling or machine learning. Nonetheless, many fundamental principles related to quality control and model testing discussed in this paper are universally applicable to the process of building models, irrespective of the modeling approach being used Kheir et al (2021). The compelling evidence of climate change's impact necessitates an active role from scientists, agronomists, and meteorologists to enhance agricultural production, advance precision forecasting, and ensure food safety, particularly in tropical regions. To address these challenges, crop simulation models provide valuable insights into the probable growth, development, and crop yield, facilitating a comprehensive assessment of soil-plant-atmosphere dynamics. One such model, the Decision Support System for Agro-Technology Transfer Model (DSSAT), is an application-driven tool designed to offer tailored recommendations for promoting agricultural sustainability. DSSAT achieves this by simulating optimal practices based on minimal experimental data provided by users, which encompasses site-specific weather data, the crop's growth period, and information about soil properties, crop management practices, and more Lucas (2014). Farm planning and management entail numerous factors, some controllable and others beyond control. In order to gain insight into these farm parameters, Mohamed et al (2016) devised a multi-period crop-mix farm model using a Linear Programming approach to assess the feasibility of decision variables.

Dury and Schaller (2012) found out that. During the growing season, farmers face the challenge of allocating their fields among various crop varieties based on past seasons' performance, current crop yields, and market prices. Additionally, they must anticipate crop production for the upcoming season. These decisions are complex and critical. To assist farmers in optimizing resource allocation, have developed a decision support model based on two key concepts: crop allocation and crop rotation decisions. Ramachandran et al (2022) argued that crop planning and allocation represent the fundamental decisions within the crop management system, as they address the intricacies inherent in the system and the available alternatives at the farm level, considering their involvement throughout various stages of crop production. (World Health Organization [WHO], 2019) made it known that Predicting crop yields is assuming greater significance due to the mounting concerns regarding food security.

Fig 1.1: Crop Selection Model

Material and Method

Let:

$Corn = q_1$
$Wheat = q_2$
$Rice = q_3$
$Soyabeans = q_4$

Table 2.1 Crop selection data

Crop	Cost of selecting crop (₦ Million)
Corn	3
Wheat	10
Rice	12
Soyabeans	14

Source: School of Agriculture Mokwa

Model Formulation

Max y = $3q_1 + 10q_2 + 12q_3 + 14q_4$ (1)

Subject to:

$q_1 + q_2 = 1$ (2)

$q_1 + q_3 = 1$ (3)

$q_2 + q_3 = 1$ (4)

$q_3 + q_4 = 1$ (5)

Where

x = binary.

Normally, Transformation 1 would be embodied in a supporting computer routine and employed to re-cast this problem into an equivalent instance of a QUBO model.

In general, the quadratic penalties to be added (for a minimization problem) are given by P

$$P\sum_{i}\left(\sum_{j=1}^{n} a_{i,j}x_{i,j} - b\right)^2 \tag{6}$$

Where,

$\sum_{i}$ is taken over all constraints in the system $Ax = b$

then

Max y' = $3q_1 + 10q_2 + 12q_3 + 14q_4$

$$+ P(q_1 + q_2)^2 + P(q_1 + q_3)^2 + P(q_2 + q_3)^2 + P(q_3 + q_4)^2 \tag{7}$$

Arbitrarily taking P to be 10, and recalling that $xj^2 = xj$ since our variables are binary, this

Becomes

$$\text{Max } y = -17q_1^2 - 10q_2^2 - 8q_3^2 - 4q_4^2 + 20q_1q_2 + 20q_1q_3 + 20q_2q_3 + 20q_3q_4 \tag{8}$$

Here, the variable q_1, q_2, q_3 *and* q_4 are binary variables that indicate whether each crop is selected (1) or not (0).

Table 2.2: **Interaction coefficients between corn, wheat, rice, and soybean**

	Corn	Wheat	Rice	Soybean
Corn	-17	20	20	0
Wheat	20	-10	20	0
Rice	20	20	-8	20
Soybean	0	0	20	-4

Using Symmetric matrix, we have:

$$y^t = \left(q^{-1}Qq\right)^t \tag{9}$$

$$= \left(q^tQq\right)^t, \ \left\{q^{-1} = q^t\right\} \tag{10}$$

$$= q^tQ^t\left(q^t\right)^t \tag{11}$$

$$= q^tQ^tq \tag{12}$$

$$= q^tQq \tag{13}$$

$$= y \tag{14}$$

Minimize $y = q^tQq$ (15)

Minimize $y = \left(q_1, q_2, q_3, q_4\right)\begin{bmatrix} -17 & 10 & 10 & 0 \\ 10 & -10 & 1 & 0 \\ 10 & 1 & -8 & 5 \\ 0 & 0 & 5 & -4 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$ (16)

Below, you'll find the results and analysis derived from implementing Python and D-Wave to solve equation (16).:

Results and Discussion

Table 3.1: QUBO Results for crop

Number	Corn	Rice	Soybean	Wheat	energy	num_oc
10	1	1	1	1	-14.0	1
5	1	1	0	1	-25.0	1
9	0	1	1	1	-18.0	1
11	1	1	1	0	-11.0	1
8	0	1	1	0	-8.0	1
6	0	1	0	1	-2.0	1
0	0	0	0	0	0.0	1
15	0	0	1	0	4.0	1
4	1	1	0	0	5.0	1
2	1	0	0	1	7.0	1
7	0	1	0	0	8.0	1
1	0	0	0	1	10.0	1
13	1	0	1	1	11.0	1
14	0	0	1	1	14.0	1
3	1	0	0	0	17.0	1
12	1	0	1	0	21.0	1

The table shows the QUBO results for a combinatorial optimization problem that involves planting different crops. The best value is the first row and shows that the best energy for the crop selection would be if we select Corn, Wheat, Rice and Soybean. The value of the energy is +41.0, after changing the sign.

Fig 3.1: **Shows Binary results for crop selection**

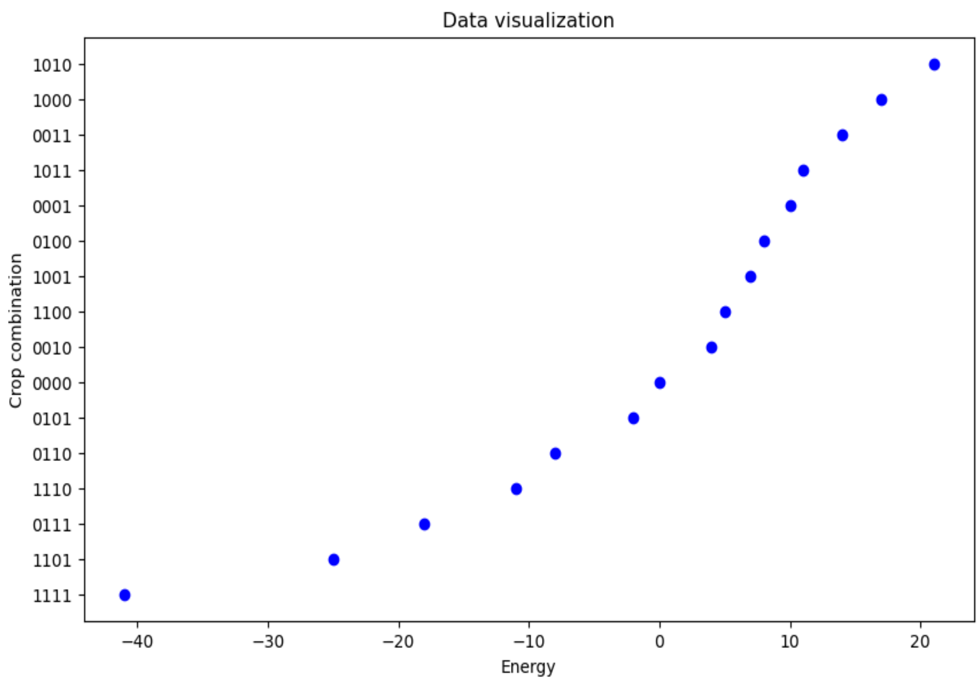


Figure 3.1 shows Binary results for crop selection it is observed from the graph that it allows for the identification of relationships between crop combinations and their energy values. For instance, it can help discern which combinations yield higher or lower energy values, or patterns in energy levels based on the presence or absence of specific crops in the combinations. Each point on this axis represents a unique combination of the four crops, identified by the binary sequence of '1's and '0's—where '1' represents the presence of a crop and '0' its absence. For example, '1101' indicates the presence of corn, rice, and wheat but the absence of soybean. This visualization provides a straightforward way to compare the energy values of different crop combinations and derive insights into how the presence or absence of specific crops contributes to the overall energy output

Crop selection Yield

The yield of a crop refers to the quantity produced per unit of land area, often measured in bushels, tons, or pounds per acre in the U.S. In a table with 16 rows, each representing different crop combinations, columns display binary values for corn, rice, soybean, and wheat, along with the energy value for each combination. Calculating the yield involves multiplying the binary value by the average yield per acre for that specific crop. For instance, the USDA reported the average yield per acre for corn in 2022 as 176.6 bushels. Thus, the yield of corn for the first row would be 1 multiplied by 176.6, resulting in a yield of 176.6 bushels per acre. Similar calculations apply to rice, soybean, and wheat, which had average yields per acre of 7,684 pounds, 51.9 bushels, and 49.7 bushels in 2022, respectively. This method allows for determining the yield of each crop in the first row.

Table 3.2: Shows Optimum crops Selection and yield results

Crop	Result (bushels per acre)
Corn	176.6
Wheat	49.7
Rice	7,684
Soybean	51.9
Total yield	7,962.2 units per acre (first row)

Table 2.1: Shows the results which represents the optimal values of the binary variables in the QUBO model,. Similarly, the yield for each crop in the remaining rows follows the same method: multiplying the binary value by the average yield per acre for the specific crop. The combined yield for the crop combination is derived by summing up the yield of each individual crop. For instance, to obtain the total yield of the first row:

Total yield: 176.6 + 7,684 + 51.9 + 49.7 = 7,962.2 units per acre

The total yield's units vary based on the specific crop, but for ease of comparison, they can be converted into a unified unit, such as tons. For instance, one bushel of corn is equal to 0.0254 tons, one pound of rice equals 0.0004536 tons, one bushel of soybean is equivalent to 0.0272 tons, and one bushel of wheat equals 0.0272 tons. Consequently, the total yield of the first row, when converted to tons, would be: Total yield: 0.0254 x 176.6 + 0.0004536 x 7,684 + 0.0272 x 51.9 + 0.0272 x 49.7 = 6.58 tons per acre

Fig 3.2: Shows Optimum crops yield results

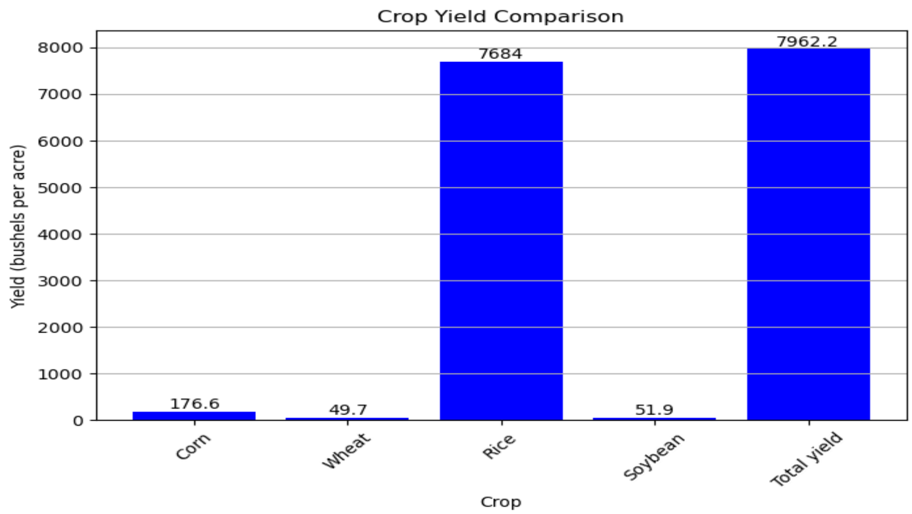
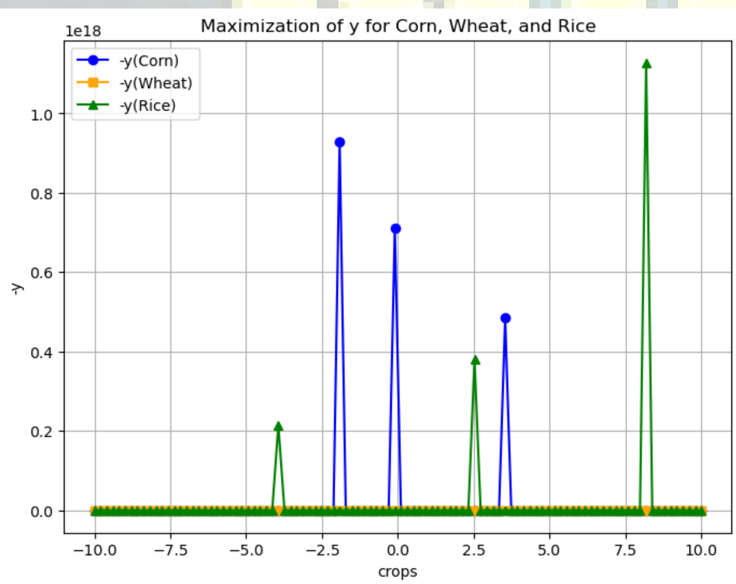


Fig 3.1: The bar chart shows that the results are positive for corn, wheat, and rice, and Soybean . This means that the optimal solution is to select corn, wheat, and rice, and soybean. The total result is 6.58tonns per acre , which is the maximum value of the objective function (yield or profit) in the QUBO model. The graph also shows that rice has the highest result, followed by wheat, corn, and soybean.

Fig 3.3: The graph of the Maximization of yield for Corn, Wheat, and Rice



The graphs show the maximization of a quadratic function $x^t Qx$, where Q a symmetric matrix is and x is a vector of variables. The function has a unique maximum value of about 200, which occurs when x is the eigenvector of Q corresponding to the largest eigenvalue. The graphs illustrate how the function value changes as one of the Crops, Corn, Wheat, or Rice, is varied, while keeping the others fixed.

Findings

(1) Optimization Approach: The study introduced a novel approach using a Quadratic Unconstrained Binary Optimization (QUBO) model to optimize crop selection and allocation in agricultural systems. This approach considered both negative impacts (costs) and positive effects (synergies) associated with different crop combinations, enabling the use of quantum annealing or classical optimization techniques to find the optimal solution.

(2) Effectiveness of the QUBO Model: Numerical experiments and case studies demonstrated the effectiveness of the QUBO-based approach. It was found that this approach provided significant improvements in crop selection and allocation decisions compared to traditional methods. It considered the complex interactions between crops and the trade-offs between costs and synergies, resulting in more efficient and profitable agricultural systems

(3) The paper presents a mathematical model for crop selection using binary variables, and it provides a cost table for selecting different crops. The model is formulated as a maximization problem, where the goal is to select a combination of crops that maximizes the objective function.

(4) The results of solving the model are shown in Table 3.1, where different crop combinations and their associated energy values are listed.

(5) Optimal Crop Selection: The best energy value corresponds to the selection of Corn, Wheat, Rice, and Soybean as the optimal crop combination.

(6) The paper calculates the yield for each crop combination by multiplying the binary values by the average yield per acre for each crop. The total yield for the optimal combination is 7,962.2 units per acre.

(7) Graphical Representation of Crop Yield: Figure 3.1 provides a bar chart illustrating the yield of each crop in the optimal combination, and it shows that the maximum total yield is 6.58 tons per acre.

(8) The chart indicates that rice has the highest yield, followed by wheat, corn, and soybean, which suggests the profitability and desirability of these crops.

Conclusion

In conclusion, the study presented a promising approach to optimize crop selection and allocation in agricultural systems using the QUBO model. The findings indicated the model's effectiveness in improving decision-making compared to traditional methods, considering the intricate dynamics of crop interactions, costs, and synergies. The flexibility and adaptability of the model make it a valuable tool for decision-makers in the agricultural sector to enhance productivity while addressing challenges such as climate change and food security. The optimal crop selection, which included corn, wheat, and rice, highlighted the potential for maximizing yield and profitability. This research contributes to the promotion of resource sustainability and efficient agricultural practices in the face of complex agricultural and environmental challenges.

Recommendations

Based on the findings and conclusions of this study, we offer the following recommendations for researchers, agricultural practitioners, and policymakers:

i. Future research should consider incorporating additional parameters such as climate variability, soil conditions, water availability, and seasonal changes into the QUBO model. This would enhance the model's predictive accuracy and practical applicability across diverse agricultural ecosystems.
ii. The QUBO-based approach should be integrated into comprehensive agricultural decision support systems that farmers can easily access. This would facilitate the practical application of these optimization techniques at the farm level.
iii. The model should be validated across different agro-ecological zones to establish its robustness and adaptability to varying environmental and climatic conditions. This would increase its reliability for global agricultural applications. Future iterations of the model should incorporate market price fluctuations and demand projections to optimize not only yield but also economic returns for farmers. This would add a crucial economic dimension to the crop selection process.

iv. Integration with Remote Sensing and IoT: Combining the QUBO model with real-time data from remote sensing technologies and IoT devices would create a dynamic crop selection system that adapts to changing conditions throughout the growing season.

v. Extension to Crop Rotation Planning: The model should be extended to multi-period planning to optimize crop rotations over several growing seasons, which would enhance soil health and sustainability while maintaining optimal yields.

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